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| EGC_Black | **YEAR 11 MATHEMATICS: SPECIALIST**  **EXTENDED PIECE OF WORK 2**  **PART A** |

**Different Number Bases**

We normally operate in base 10 for all of our arithmetic but in various other circumstances it may be advantageous to use a different number base. Bases commonly used are base 2 (binary) and base 16 (hexadecimal).

In base 10, there are 10 digits (0, 1, 2, …) with 9 being the biggest.

When writing numbers we use ‘columns’, which from the ‘right’ are : 1’s, 10’s, 100’s ….

Other number bases operate similarly.

Example:

In base 4, there will be four digits, with 3 being the largest (0, 1, 2 and 3) .

The ‘columns’ will be : 1’s, 4’s, 16’s, 64’s, ….

For notational purposes if a number is written in a base other than 10, the base might be shown as a subscript.

Thus : 234 = 2 x 4 + 3 x 1 = 11 (in base 10)

Similarly: 30124 = 3 x 64 + 0 x 16 + 1 x 4 + 2 x 1 = 198

**Practice Questions: A**

**1.** Change the following numbers to their base 10 equivalent:

(a) 3024 (b) 110102 (c) 30245

(d) 61247 (e) 213215 (f) 213214

(g) 213216 (h) 213217 (i) 2132116

**2.** Change the number 103 (base 10) to its equivalent in the given base:

(a) base 3 (b) base 4 (c) base 5

(d) base 9 (e) base 2

## Addition (and the other operations)

When adding numbers in base 10, we add ‘by columns’ (starting on the right); if the result is greater than 9 (the largest possible digit) then we must ‘carry’. The process is similar when operating in other bases.

**Examples**

**(Base 4)** 133

+ 2022

2221

(NB 3+2 = 5 ; 5, in base 4, is 11; put down 1 and carry 1;

3+2+1 = 6; 6 in base 4 is 12; put down 2 and carry 1; )

**(Base 5)** 2304 (NB: 4+3+3=10; 10 in base 5 = 20; put down 0, carry 2

3423 3+4+2 = 9 ; 9 in base 5 = 14; put down 4, carry 1

+ 203 2+3+1 = 6 ; 6 in base 5 = 11 )

11440

Subtraction, multiplication and division are carried out in the same manner as in base 10

**(Base 6)** 2315 (NB: 5 – 2 = 3; 1 – 3 cannot do, so borrow a 1 from the 3,

- 432 116 = 7, so 7 – 3 = 4; now it is 2 – 4 (because you borrowed

1443 1 so borrow 1 from the 2, 126 = 8, so 8 – 4 = 4; 1 – 0 = 1)

**(Base 7)** 532 (NB: 4 x 2 = 8 = 117; put down 1 carry 1; 4 x 3 + 1 = 13 = 167;

x 34 put down 6 carry 1; 4 x 5 + 1 = 21 = 307; put down 0; 3 x 2 = 6

3061 3 x 3 = 9 = 127; put down 2 carry 1; 3 x 5 + 1 = 16 = 227;

22260 1 + 0 = 1; 6 + 6 = 12 = 157, put down 5 carry 1; 1 + 0 + 2 = 3;

25351 3 + 2 = 5; 2 out the front)

**(Base 8)** 412 ÷ 7 (NB: 7 doesn’t go into 4. 418 = 33; 7 into 33 goes 4 r 5

put down the 4 ‘carry’ the 5; 528 = 42, 7 into 42 goes 6 times.

**Practice Questions: B**

**3.** Perform the following additions in the indicated base

a) 253 + 443 (base 6) b) 253 + 443 (base 7)

(c) 253 + 443 (base 8) (d) 110 + 10010 + 1110 (base 2)

(e) 110 + 10010 + 1110 (base 3)

(f) 332 + 23100 + 23121 + 20023 (base 4)

(g) 332 + 23100 + 23121 + 20023 (base 5)

**4.** Perform the following subtractions in the indicated base

(a) 3214 – 332 (base 5) (b) 12210 – 211 (base 3)

(c) 55023 – 40541 (base 6)

**5.** Perform the following multiplications in the indicated base

(a) 3123 x 22 (base 4) (b) 3123 x 22 (base 5)

(c) 3123 x 22 (base 7) (d) 110121 x 221 (base 3)

(e) 286 x 77 (base 9)

**6.** Perform the following divisions in the indicated base

(a) 2202 ÷ 12 (base 4) (b) 3411 ÷ 23 (base 5)

(c) 4354 ÷ 25 (base 7) (d) 201202 ÷ 111 (base 3)

**Bases larger than 10**

When operating in bases larger than 10, more than 10 ‘digits’ are needed. Otherwise the methods are identical as for base 10 or any smaller base.

Example : Base 12; (12 ‘digits’ needed, the largest being ‘11’ ; use ‘a’ for 10, and ‘b’ for 11 )

Thus 7512 = 7 x 12 + 5 x 1 = 89 in base 10

Similarly: 8a712 = 8 x 144 + 10 x 12 + 7 x 1 = 1279 in base 10

Similarly: 30ba12 = 3 x 123 + 0 x 122 + 11 x 12 + 10 = 5326 in base 10

**Practice Questions: C**

**7.** Perform the following operations in the indicated base

(a) 279 + 756 + 9b (base 12) (b) 6a0 – 25b (base 12)

(c) 784 x 95 (base 12)

(d) 6c89 + 82b + a0958 (base 13 : a = 10 , b = 11 , c = 12)

(e) 9a7d x b8 (base 16: a = 10, b = 11 , c = 12 , d = 13 , e = 14 , f = 15)

(f) bcd8 + 974 + 1005d (base 16)

**Decimals**

As in base 10 (where digits after the decimal place correspond to the ‘columns’ 10-1, 10-2, … ) we can use decimals in other bases.

Thus, in base 5 :

21.34 is equivalent to 2 x 5 + 1 x 1 + 3 x 5-1 + 4 x 5-2 = 11 + 3/5 + 4/25 = 11 19/25

**8.** Change the following numbers to their base 10 equivalent (you may leave as a fraction)

(a) 3.256 (b) 12.2213 (c) 17.238

(d) 101.11012 (e) 21.667 (f) 32.0314

**ANSWERS**

**A** 1 (a) 50 (b) 26 (c) 389 (d) 2125 (e) 1461 (f) 633 (g) 2929 (h) 5307 (i) 135969

2 (a) 10211 (b) 1213 (c) 403 (d) 124 (e) 1100111

**B** 3 (a) 1140 (b) 1026 (c) 716 (d) 100110 (e) 12000 (f) 133302 (g) 122131

4 (a) 2332 (b) 11222 (c) 14042

5 (a) 202032 (b) 124311 (c) 102036 (d) 102122211 (e) 25036

6 (a) 123 (b) 122 (c) 145 (d) 1112

**C** 7 (a) ab2 (b) 441 (c) 60558 (d) a8442 (e) 6f09d8 (f) 1c6a9

8 (a) (b) (c) (d) (e) (f)